IMPULSIVE STABILIZATION OF LARGE-SCALE DYNAMICAL SYSTEMS

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ABSTRACT. In this paper, we consider impulsive large-scale systems without delay and with delays. By establishing an exponential estimate for differential inequality, following the idea of vector Lyapunov's approach and employing the formula for the variation of parameters, we obtain the criteria on uniform stability, global asymptotical stability and global exponential stability of the impulsive large-scale systems. Based on the stability criteria, we propose the simple method to stabilize the large-scale systems by utilizing impulsive effects. Explicit steps on impulsive stabilization of the systems are presented. Examples and simulations are given to illustrate the effectiveness of the results.

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1. INTRODUCTION

Impulsive dynamical systems have attracted increasing interest since impulsive effects widely exist in many dynamical systems involving such fields as population dynamics, automatic control, drug administration, communication networks and so on. The fundamental theoretics and systemic method of impulsive dynamical systems have been established in the recent years, see [1]-[3]. On the other hand, the dynamical systems that people are faced with are becoming more and more complicated in structure and large in scale. In the past years, large-scale dynamical systems have been intensively investigated and some popular tools such as Lyapunov method and comparison method have been applied successfully to study the dynamics of large-scale systems ([4]-[7]). A large number of the criteria on the stability have been derived for impulsive systems and large-scale systems, respectively (see, e.g., Refs. [1]-[9]). On the basis of these theoretics and method, it is natural to further investigate the stability of impulsive large-scale dynamical systems([10], [11], etc.).

Stabilization of dynamical systems is an important subject in both theoretic research and engineering applications. By employing continuous state feedback mechanism, the approach of stabilization has been well developed in various dynamical systems (see, [6],[11], etc.). Recently, impulsive stabilization of dynamical systems has become another important approach since it may be simpler and cheaper in implementation mechanisms([10], [12]-[15]). The main difficulty in using impulsive stabilization comes from the requirement that the continuous portion in impulsive systems must not be stable and the systems are stabilized only by utilizing impulsive effects. Impulsive stabilization of large-scale dynamical systems with delays may be more difficult and few results in this direction have been reported.

In this paper, we consider impulsive large-scale systems without delay and with delays. By establishing an exponential estimate for differential inequality and combining the idea of vector Lyapunov's approach, we analyze the stability of the zero solution of the impulsive system without delay. Furthermore, by using the formula for the variation of parameters and estimating the Cauchy matrix of the isolated subsystems, we obtain the stability of the zero solution of the impulsive large-scale system with delays. It is important that our results don't require the stability of the corresponding continuous system. Consequently, our criteria can be easily applied to stabilize the dynamical systems by employing impulsive effects and simple steps of impulsive stabilization are provided. Examples and stimulations are given to illustrate the feasibility and effectiveness of our approach.

2. PRELIMINARIES

Let $\mathbb{N}$ be the natural numerical set, $\mathbb{R}^n$ be the space of n-dimensional real column vectors and $\mathbb{R}^{m\times n}$ be the set of $m \times n$ real matrices. Denote $\mathbb{R}^+ = [0, \infty)$ and $a^+ = \max\{0, a\}$ for $a \in \mathbb{R}$. E denotes the unit matrix.

Let $\tau \geq 0$ and $t_1 < \cdots < t_k < t_{k+1} < \ldots (k \in \mathbb{N})$ be the fixed impulsive moments with $\lim\limits_{k\to\infty} t_k = \infty$. For $\phi = (\phi_1, \ldots, \phi_n)^T : \mathbb{R} \to \mathbb{R}^l, l \in \mathbb{N}$, denote

$$\phi(t^+) = \lim_{s\to 0^+} \phi(t+s),\ \phi(t^-) = \lim_{s\to 0^-} \phi(t+s),$$
$$D^+\phi(t) = \limsup_{s\to 0+0} \tfrac{\phi(t+s)-\phi(t)}{s},\ [\phi_i(t)]_\tau = \sup_{-\tau\leq s\leq 0} \{\phi_i(t+s)\},$$

where $D^+\phi(t)$ is called the upper right derivative of $\phi(t)$ (see Yoshizawa[17]).

$C[X, Y]$ denotes the space of continuous mappings from the topological space X to the topological space Y.

$PC[I, \Omega] := \{\phi : I \to \Omega |\ \phi(t^+) = \phi(t)$ for $t \in I$, $\phi(t^-)$ exists for $t \in I$, $\phi(t^-) = \phi(t)$ for all but points $t_k \in I\}$, where the interval $I \subset \mathbb{R}$, and the region $\Omega \subset \mathbb{R}^l$ or $\Omega \subset \mathbb{R}^{l\times m}$, $l, m \in \mathbb{N}$.

$PC := \{\phi : [-\tau, 0] \to \mathbb{R}^n \mid \phi(t^+) = \phi(t)$ for $t \in [-\tau, 0)$, $\phi(t^-)$ exists for $t \in (-\tau, 0]$, $\phi(t^-) = \phi(t)$ for all but at most a finite number of points $t \in (-\tau, 0]\}$.

For $x = (x_1, \dots, x_n)^T \in \mathbb{R}^n$, $A = (a_{ij}) \in \mathbb{R}^{n\times n}$, $\phi \in PC$, we introduce the following norms, respectively,

$$\|x\| = \sum_{j=1}^{n} |x_j|, \quad \|A\| = \max_{1\le j\le n} \sum_{i=1}^{n} |a_{ij}|, \quad \|\phi\|_\tau = \sup_{-\tau\le s\le 0} \{\|\phi(s)\|\}.$$

The following lemma is a modification of the continuous delay differential inequality (see Xu [5]) and will play an important role in this paper.

Lemma 2.1. *Let $u_i(t) \in C[[\sigma, b), \mathbb{R}^n]$ be a solution of the differential inequalities*

$$\begin{cases} D^+u_i(t) \le \sum\limits_{j=1}^{n} p_{ij}(t)u_j(t) + \sum\limits_{j=1}^{n} q_{ij}(t)[u_j(t)]_\tau, \ t \in [\sigma, b), \\ u(\sigma + s) = (u_1(\sigma + s), \dots, u_n(\sigma + s))^T \in PC, \ s \in [-\tau, 0], \end{cases} \tag{2.1}$$

where $\tau \ge 0$, $\sigma < b \le +\infty$, $p_{ij}(t) \ge 0$ for $i \ne j$, $q_{ij}(t) \ge 0$ and $i, j = 1, 2, \dots, n$. Suppose that there exists an integrable function $r(t), t \in [\sigma - \tau, b)$ such that

$$\sum_{j=1}^{n} p_{ij}(t) + e^{\sup\limits_{\theta\in[-\tau,0]}\{\int_{t+\theta}^{t} r(s)ds\}} \sum_{j=1}^{n} q_{ij}(t) < -r(t), \ t \in [\sigma, b). \tag{2.2}$$

If the initial condition satisfies

$$u_i(t) \le \kappa e^{-\int_\sigma^t r(s)ds}, \quad \kappa \ge 0, \ t \in [\sigma - \tau, \sigma], \ i = 1, \dots, n, \tag{2.3}$$

then for $i = 1, \dots, n$

$$u_i(t) \le \kappa e^{-\int_\sigma^t r(s)ds}, \ t \in [\sigma, b). \tag{2.4}$$

Proof. We first prove that for any number $\epsilon > 0$

$$u_i(t) \le (\kappa + \epsilon)e^{-\int_\sigma^t r(s)ds} \triangleq y(t), \ i = 1, \dots, n, t \in [\sigma, b). \tag{2.5}$$

Let

$$\mathcal{J} = \{i | u_i(t) > y(t) \text{ for some } t \in [\sigma, b)\},$$
$$\theta_i = \inf\{t \in [\sigma, b) | u_i(t) > y(t), i \in I\}.$$

If the inequality (2.5) is not true, then $\mathcal{J}$ is a nonempty set and there must exist some integer $m \in \mathcal{J}$ such that $\theta_m = \min\limits_{i\in I}\{\theta_i\} \in [\sigma, b)$. Employing the continuity of functions $u_i(t)$ and $y_i(t)$ for $t \in [\sigma, b), i = 1, \dots, n$, from (2.3), we can get

$$u_i(t) \le y(t), \quad \sigma - \tau \le t \le \theta_m, \ i = 1, \dots, n, \tag{2.6}$$

$$u_m(\theta_m) = y(\theta_m), \ D^+u_m(\theta_m) \ge \dot{y}(\theta_m). \tag{2.7}$$

Combining with

$$[y(\theta_m)]_\tau = (\kappa + \epsilon) \sup_{\theta\in[-\tau,0]} \{e^{-\int_\sigma^{\theta_m+\theta} r(s)ds}\} = (\kappa + \epsilon)e^{-\int_\sigma^{\theta_m} r(s)ds} \sup_{\theta\in[-\tau,0]} \{e^{\int_{\theta_m+\theta}^{\theta_m} r(s)ds}\},$$

we have

$$
\begin{aligned}
D^+u_m(\theta_m) &\le \sum_{j=1}^{n}[p_{mj}(\theta_m)u_j(\theta_m)+q_{mj}(\theta_m)[u_j(\theta_m)]_\tau] \\
&\le \sum_{j=1}^{n}[p_{mj}(\theta_m)y(\theta_m)+q_{mj}(\theta_m)[y(\theta_m)]_\tau] \\
&= \sum_{j=1}^{n}[p_{mj}(\theta_m)+q_{mj}(\theta_m)e^{\sup\limits_{\theta\in[-\tau,0]}\{\int_{\theta_m+\theta}^{\theta_m} r(s)ds\}}](\kappa+\epsilon)e^{-\int_\sigma^{\theta_m} r(s)ds} \\
&< -r(\theta_m)(\kappa+\epsilon)e^{-\int_\sigma^{\theta_m} r(s)ds} \\
&= \dot{y}(\theta_m),
\end{aligned}
$$

which contradicts the inequality in (2.7). Then, (2.5) is true for any $\epsilon > 0$. Letting $\epsilon \to 0^+$, we obtain the estimate (2.4). □

Remark 2.1. When $\tau = 0$ and $q_{ij}(t) = 0$, the above conditions (2.1), (2.2) and (2.3) can be written as follows, respectively,

$$
\begin{aligned}
&D^+u_i(t) \le \sum_{j=1}^{n} p_{ij}(t)u_j(t),\ t\in[\sigma,b); \\
&\sum_{j=1}^{n} p_{ij}(t) < -r(t),\ t\in[\sigma,b); \\
&u_i(\sigma)\le\kappa, \kappa\ge 0, i=1,\dots,n.
\end{aligned}
$$

For the definition of uniform stability, asymptotic stability and exponential stability, ones can refer to [1], [8], [18].

3. IMPULSIVE LARGE-SCALE SYSTEMS WITHOUT DELAY

Consider an impulsive large-scale dynamical system without delay

$$
\begin{cases}
\dot{x}_i(t) = A_i(t)x_i(t) + f_i(t,x(t)), & t\neq t_k,\ i=1,\dots,m \\
\triangle x_i(t) = B_{ik}x_i(t^-) + I_i(t,x(t^-)), & t=t_k,\ k\in\mathbb{N},
\end{cases} \tag{3.1}
$$

where $x_i = (x_1^{(i)},\dots,x_{n_i}^{(i)})^T \in \mathbb{R}^{n_i}$, $\sum\limits_{i=1}^{m} n_i = n$, $x = (x_1^T,\dots,\ x_m^T)^T \in \mathbb{R}^n$, $\dot{x}_i$ is the right-hand derivative, $\triangle x_i(t) = x_i(t^+) - x_i(t^-)$, $A_i(t) = (a_{lj}^{(i)}(t)) \in PC[\mathbb{R},\mathbb{R}^{n_i\times n_i}]$, $f_i = (f_1^{(i)},\dots,f_{n_i}^{(i)})^T \in C[[t_{k-1},t_k)\times\mathbb{R}^n,\mathbb{R}^{n_i}]$, $B_{ik}\in\mathbb{R}^{n_i\times n_i}$, $I_i \in C[\mathbb{R}\times\mathbb{R}^n,\mathbb{R}^{n_i}]$, $f_i(t,0)=0, I_i(t,0)=0$.

A function $x(t) : [t_0,+\infty) \to \mathbb{R}^n$ is called a solution of Eq.(3.1) with the initial condition given by

$$
x(t_0) = x_0 \in \mathbb{R}^n, \tag{3.2}
$$

if $x(t)$ is continuous at $t \neq t_k$ and $t \geq t_0$, $x(t_k) = x(t_k^+)$ and $x(t_k^-)$ exists, $x(t)$ satisfies Eq.(3.1) for $t \geq t_0$ under the initial condition.

According to [1], the initial-value problem (3.1), (3.2) has the unique solution $x(t, t_0, x_0)$ if f_i, I_i satisfy the following conditions.

(H_1) There exist $l_{ij}(t) \in PC[\mathbb{R}, \mathbb{R}^+]$, $i, j = 1, \dots, m$ such that

$$\|f_i(t,x)\| \leq \sum_{j=1}^{m} l_{ij}(t)\|x_j\|, \ \ \forall x \in \mathbb{R}^n, t \in R.$$

(H_2) There exist $u_{ij}(t) \in C[\mathbb{R}, \mathbb{R}^+], i, j = 1, \dots, m$ such that

$$\|I_i(t,x)\| \leq \sum_{j=1}^{m} u_{ij}(t)\|x_j\|, \ \ \forall x \in \mathbb{R}^n, t \in R.$$

For convenience, we denote

$$\mu_i(t) = \max_{1 \leq l \leq n_i}\{a_{ll}^{(i)}(t) + \sum_{j=1, j\neq l}^{n_i} |a_{jl}^{(i)}(t)|\}, i = 1, \dots, m, \tag{3.3}$$

$$p(t) = \max_{1 \leq i \leq m}\{\mu_i(t) + \sum_{j=1}^{m} l_{ij}(t)\}, \tag{3.4}$$

$$\eta_k = \max_{1 \leq i \leq m}\{\|E + B_{ik}\| \ + \sum_{j=1}^{m} u_{ij}(t_k)\}, k \in \mathbb{N}. \tag{3.5}$$

Theorem 3.1. *Assume that (H_1) and (H_2) hold. Let $\sup_{k\in\mathbb{N}}\{\int_{t_{k-1}}^{t_k} p^+(s)ds\} < \infty$. Then,*

i) the zero solution of Eq.(3.1) is uniformly stable if

$$\ln \eta_k \leq -\int_{t_{k-1}}^{t_k} p(s)ds, \ k \in \mathbb{N}; \tag{3.6}$$

ii) the zero solution of Eq.(3.1) is globally asymptotically stable if

$$\overline{\lim_{k\to\infty}}\{\ln \eta_k + \int_{t_{k-1}}^{t_k} p(s)ds\} < 0, \tag{3.7}$$

$$\sup_{k\in\mathbb{N}}\{t_k - t_{k-1}\} < \infty; \tag{3.8}$$

iii) the zero solution of Eq.(3.1) is globally exponentially stable if

$$\sup_{k\in\mathbb{N}}\{\ln \eta_k + \int_{t_{k-1}}^{t_k} p(s)ds\} < 0, \tag{3.9}$$

$$\sup_{k\in\mathbb{N}}\{t_k - t_{k-1}\} < \infty. \tag{3.10}$$

Proof. For any $x_0 \in \mathbb{R}^n$, let $x(t) = (x_1(t), \dots, x_m(t))^T$ be a solution through (t_0, x_0), where $x_i \in \mathbb{R}^{n_i}$. No loss of generality, we assume $t_0 < t_1$. Define $V(t) = (V_1(t), \dots, V_m(t))^T$, where

$$V_i(t) = \|x_i\| = \sum_{l=1}^{n_i} |x_l^{(i)}(t)|, \ i = 1, \dots, m.$$

Calculating the upper right derivative D^+V_i along the solution $x(t)$ of Eq.(3.1), from (H_1), we have

$$(3.11)\quad \begin{aligned} D^+V_i(t) &= \sum_{l=1}^{n_i} \mathrm{sgn}(x_l^{(i)}(t))\dot{x}_l^{(i)}(t) \\ &= \sum_{l=1}^{n_i} \mathrm{sgn}(x_l^{(i)}(t))[\sum_{j=1}^{n_i} a_{lj}^{(i)}(t)x_j^{(i)}(t) + f_l^{(i)}(t, x(t))] \\ &\leq \sum_{l=1}^{n_i}[a_{ll}^{(i)}(t)|x_l^{(i)}(t)| + \sum_{j=1, j\neq l}^{n_i} |a_{lj}^{(i)}(t)|\ |x_j^{(i)}(t)| + |f_l^{(i)}(t, x(t))|] \\ &= \sum_{l=1}^{n_i}[a_{ll}^{(i)}(t) + \sum_{j=1, j\neq l}^{n_i} |a_{jl}^{(i)}(t)|]\ |x_l^{(i)}(t)| + \|f_i(t, x(t))\| \\ &\leq \mu_i(t)\|x_i\| + \sum_{j=1}^{m} l_{ij}(t)\|x_j\| \\ &= \mu_i(t)V_i + \sum_{j=1}^{m} l_{ij}(t)V_j, \quad t \in [t_{k-1}, t_k),\ k \in \mathbb{N}. \end{aligned}$$

For any number $\epsilon > 0$,

$$(3.12)\qquad \mu_i(t) + \sum_{j=1}^{m} l_{ij}(t) \leq p(t) < p(t) + \epsilon,\ i = 1, \ldots, m.$$

Let $\eta_0 = 1$. In the following, by the induction, we shall prove that for $i = 1, \ldots, m$

$$(3.13)\qquad V_i(t) \leq \eta_0 \ldots \eta_{k-1}\|V(t_0)\|e^{\int_{t_0}^{t}(p(s)+\epsilon)ds}, t_{k-1} \leq t < t_k, k \in \mathbb{N}.$$

Combining (3.11), (3.12) and Lemma 2.1, we obtain

$$(3.14)\qquad V_i(t) \leq \eta_0\|V(t_0)\|e^{\int_{t_0}^{t}(p(s)+\epsilon)ds},\ i = 1, \ldots, m, t_0 \leq t < t_1.$$

Suppose that

$$(3.15)\qquad V_i(t) \leq \eta_0 \ldots \eta_{k-1}\|V(t_0)\|e^{\int_{t_0}^{t}(p(s)+\epsilon)ds}, i = 1, \ldots, m, t_{k-1} \leq t < t_k.$$

From Condition (H_2), then

$$(3.16)\quad \begin{aligned} V_i(t_k) &= \|x_i(t_k)\| = \|(E + B_{ik})x_i(t_k^-) + I_i(t_k, x(t_k^-))\| \\ &\leq \|E + B_{ik}\|\ \|x_i(t_k^-)\| + \sum_{j=1}^{m} u_{ij}(t_k)\|x_j(t_k^-)\| \\ &\leq \{\|E + B_{ik}\|\ + \sum_{j=1}^{m} u_{ij}(t_k)\}\eta_0 \ldots \eta_{k-1}\|V(t_0)\|e^{\int_{t_0}^{t_k}(p(s)+\epsilon)ds} \\ &\leq \eta_0 \ldots \eta_{k-1}\eta_k\|V(t_0)\|e^{\int_{t_0}^{t_k}(p(s)+\epsilon)ds}. \end{aligned}$$

Employing (3.11), (3.12), (3.16) and Lemma 2.1, we have

$$V_i(t) \leq \eta_0 \ldots \eta_k\|V(t_0)\|e^{\int_{t_0}^{t}(p(s)+\epsilon)ds}, i = 1, \ldots, m, t_k \leq t < t_{k+1}.$$

Therefore, we obtain the estimate (3.13). Letting $\epsilon \to 0^+$, then for $i = 1, \ldots, m$

$$(3.17) \qquad V_i(t) \leq \eta_0 \ldots \eta_{k-1} \|V(t_0)\| e^{\int_{t_0}^t p(s)ds}, t_{k-1} \leq t < t_k, k \in \mathbb{N}.$$

i) Let $\varrho := \sup_{k \in \mathbb{N}} \{\int_{t_{k-1}}^{t_k} p^+(s)ds\} < \infty$. From (3.6) and (3.17), we can get

$$\begin{aligned} V_i(t) &\leq \eta_0 \eta_1 \ldots \eta_{k-1} \|V(t_0)\| e^{\int_{t_0}^t p(s)ds} \\ &\leq e^{-\int_{t_0}^{t_{k-1}} p(s)ds} e^{\int_{t_0}^t p(s)ds} \|V(t_0)\| \\ &= e^{\int_{t_{k-1}}^t p(s)ds} \|V(t_0)\| \\ &\leq e^{\int_{t_{k-1}}^{t_k} p^+(s)ds} \|V(t_0)\| \\ &\leq e^{\varrho} \|V(t_0)\|, \ i = 1, \ldots, m, t_{k-1} \leq t < t_k, k \in \mathbb{N}, \end{aligned}$$

which implies the zero solution of Eq.(3.1) is uniformly stable.

ii) Let $0 < \rho := \sup_{k \in \mathbb{N}} \{t_k - t_{k-1}\} < \infty$. From the strict inequality (3.7), there must be a $\lambda_0 > 0$ and a positive integer n_0 such that for $k \geq n_0$

$$\ln \eta_k + \int_{t_{k-1}}^{t_k} p(s)ds \leq -\lambda_0 < 0.$$

Thus, for $k \geq n_0$

$$\eta_k \leq e^{-\int_{t_{k-1}}^{t_k} p(s)ds - \lambda_0} \leq e^{-\int_{t_{k-1}}^{t_k} [p(s) + \lambda_0/\rho]ds}.$$

Combining with (3.17), then for $k \geq n_0 + 1, t_{k-1} \leq t < t_k$

$$\begin{aligned} V_i(t) &\leq c_0 \ \eta_{n_0} \ldots \eta_{k-1} e^{\int_{t_{n_0-1}}^t p(s)ds} \|V(t_0)\| \\ &\leq c_0 \ e^{-\int_{t_{n_0-1}}^{t_{k-1}} [p(s)+\lambda_0/\rho]ds} e^{\int_{t_{n_0-1}}^t p(s)ds} \|V(t_0)\| \\ &= c_0 \ e^{\int_{t_{k-1}}^t [p(s)+\lambda_0/\rho]ds} e^{-\int_{t_{n_0-1}}^t \lambda_0/\rho ds} \|V(t_0)\| \\ &\leq c_0 \ e^{\int_{t_{k-1}}^{t_k} p^+(s)ds + (\lambda_0/\rho)(t_k - t_{k-1})} e^{-(\lambda_0/\rho)(t - t_{n_0-1})} \|V(t_0)\| \\ &\leq c \, e^{-(\lambda_0/\rho)(t - t_{n_0-1})} \|V(t_0)\|, \ i = 1, \ldots, m, \end{aligned}$$

where $c_0 := \eta_0 \eta_1 \ldots \eta_{n_0-1} e^{\int_{t_0}^{t_{n_0-1}} p(s)ds}$ and $c := c_0 e^{\varrho + \lambda_0}$. Hence, the zero solution of Eq.(3.1) is globally asymptotically stable since $ce^{-(\lambda_0/\rho)(t - t_{n_0-1})} \to 0$ as $t \to +\infty$.

iii) Let $\sup_{k \in \mathbb{N}} \{\ln \eta_k + \int_{t_{k-1}}^{t_k} p(s)ds\} = -\lambda < 0$. In a similar way as the case ii), we can obtain for $i = 1, \ldots, m$

$$V_i(t) \leq e^{\varrho + \lambda} e^{-(\lambda/\rho)(t - t_0)} \|V(t_0)\|, \ t_{k-1} \leq t < t_k, k \in \mathbb{N},$$

which implies the zero solution of Eq.(3.1) is globally exponentially stable. □

Based on the above stability theorem, we easily obtain the following criteria on the stabilization of the large-scale dynamical system

$$(3.18) \qquad \dot{x}_i(t) = A_i(t) x_i(t) + f_i(t, x(t)), i = 1, \ldots, m,$$

by utilizing the linear impulses

$$\triangle x_i(t_k) = x_i(t_k) - x_i(t_k^-) = B_{ik}x_i(t_k^-),\ k \in \mathbb{N}. \tag{3.19}$$

Theorem 3.2. *Let (H_1) hold. If*

$$\sup_{k\in\mathbb{N}}\{t_k - t_{k-1}\} < \infty,\ \sup_{k\in\mathbb{N}}\{\int_{t_{k-1}}^{t_k} p^+(s)ds\} < \infty,$$

and there exists a constant $\gamma > 1$ $(\gamma \geq 1)$ such that

$$\ln(\gamma\|E + B_{ik}\|) + \int_{t_{k-1}}^{t_k} p(s)ds \leq 0,\ k \in \mathbb{N},$$

then the zero solution of (3.18), (3.19) is globally exponentially stable (uniformly stable).

Remark 3.1. According to Theorem 3.2, we easily present some explicit steps of impulsive stabilization (e.g., uniformly stable) for the large-scale system (3.18).

1) Choose the impulsive moments $\{t_k\}$ such that $\int_{t_{k-1}}^{t_k} p^+(s)ds < \infty, k \in \mathbb{N}$. Especially, we may take $t_k - t_{k-1} \equiv \rho$ if $p(t)$ is a bounded function.
2) Calculate the integral $\int_{t_{k-1}}^{t_k} p(s)ds$.
3) Choose impulsive matrices $B_{ik}, k \in \mathbb{N}$ satisfying $\|E+B_{ik}\| \leq e^{-\int_{t_{k-1}}^{t_k} p(s)ds}$.

From the proof process of Theorem 3.1, we also obtain the estimate for the Cauchy matrix of the isolated subsystem. For the definition and properties of the Cauchy matrix, we can refer to [1], [2, p.18].

Theorem 3.3. *Let $C_i(t, t_0)$ be the Cauchy matrix of the linear system*

$$\begin{cases} \dot{x}_i(t) = A_i(t)x_i(t), & t \neq t_k,\ i = 1, \ldots, m, \\ \triangle x_i(t) = B_{ik}x_i(t^-), & t = t_k,\ k \in \mathbb{N}, \end{cases} \tag{3.20}$$

and

$$\varrho_i := \sup_{k\in\mathbb{N}} \int_{t_{k-1}}^{t_k} \mu_i^+(s)ds < \infty, \ \ \textit{where}\ \ \mu_i(t) = \max_{1\leq l\leq n_i}\{a_{ll}^{(i)}(t) + \sum_{j=1,j\neq l}^{n_i} |a_{jl}^{(i)}(t)|\}.$$

If

$$\ln\|E + B_{ik}\| + \int_{t_{k-1}}^{t_k} \mu_i(s)ds \leq -\lambda_i \leq 0,\ \ t_k - t_{k-1} \leq \rho,$$

then,

$$\|C_i(t, t_0)\| \leq c_i e^{-(\lambda_i/\rho)(t-t_0)},\ \textit{where}\ \ c_i := e^{\varrho_i+\lambda_i}, t \geq t_0.$$

Furthermore, the zero solution of (3.20) is globally exponentially stable (uniformly stable) provided that $\lambda_i > 0$ $(\lambda_i \geq 0)$.

4. IMPULSIVE LARGE-SCALE SYSTEMS WITH DELAYS

Consider an impulsive large-scale dynamical system with delays

$$(4.1)\quad \begin{cases} \dot{x}_i(t) = A_i(t)x_i(t) + g_i(t, x(t), x(t-\tau_i(t))), & t \neq t_k,\ i = 1, \dots, m \\ \triangle x_i(t) = B_{ik}x_i(t) + I_i(t, x(t^-)), & t = t_k,\ k \in \mathbb{N}, \end{cases}$$

where $A_i(t) = (a_{lj}^{(i)}(t)) \in PC[\mathbb{R}, \mathbb{R}^{n_i\times n_i}]$, $x_i = (x_1^{(i)}, \dots, x_{n_i}^{(i)})^T \in \mathbb{R}^{n_i}$, $x = (x_1^T, \dots, x_m^T)^T \in \mathbb{R}^n$, $g_i = (g_1^{(i)}, \dots, g_{n_i}^{(i)})^T \in C[[t_{k-1}, t_k) \times \mathbb{R}^n \times \mathbb{R}^n, \mathbb{R}^{n_i}]$, $B_{ik} \in \mathbb{R}^{n_i\times n_i}$, $I_i \in C[\mathbb{R} \times \mathbb{R}^n, \mathbb{R}^{n_i}]$, $0 \leq \tau_i(t) \leq \tau$ $(\tau > 0)$, $\sum_{i=1}^{m} n_i = n$, $g_i(t, 0, 0) = 0$, $I_i(t, 0) = 0$.

A function $x(t) : [t_0 - \tau, +\infty) \to \mathbb{R}^n$ is called a solution of Eq.(4.1) with the initial condition given by

$$(4.2)\qquad x(t_0 + s) = \phi(s) \in PC,\ s \in [-\tau, 0],$$

if $x(t)$ is continuous at $t \neq t_k$ and $t \geq t_0$, $x(t_k) = x(t_k^+)$ and $x(t_k^-)$ exists, $x(t)$ satisfies Eq.(4.1) for $t \geq t_0$ under the initial condition.

According to [16], the initial-value problem (4.1), (4.2) has the unique solution $x(t, \sigma, \phi)$ if g_i, I_i satisfy the conditions (H_1') and (H_2).

(H_1') There exist $l_{ij}(t), q_{ij}(t) \in PC[\mathbb{R}, \mathbb{R}^+]$, $i, j = 1, \dots, m$ such that

$$\|g_i(t, x, y)\| \leq \sum_{j=1}^{m} l_{ij}(t)\|x_j\| + \sum_{j=1}^{m} q_{ij}(t)\|y_j\|,\ \ \forall x \in \mathbb{R}^n, y \in \mathbb{R}^n,\ t \in R.$$

Theorem 4.1. *Assume that, in addition to (H_1') and (H_2), the following conditions are satisfied.*

(H_3) There exist constants $c_i \geq 1$ and functions $p_i(t) \in PC[\mathbb{R}, \mathbb{R}^+]$ such that

$$\|C_i(t, t_0)\| \leq c_i e^{-\int_{t_0}^{t} p_i(s)ds}, t \geq t_0,$$

where $C_i(t, t_0)$ are Cauchy matrices of the isolated subsystems (3.20).

(H_4) Let $\gamma \geq 1$ and $\alpha(t) \in PC[\mathbb{R}, \mathbb{R}^+]$ satisfy

$$-p_i(t) + \sum_{j=1}^{m} c_i l_{ij}(t) + \gamma \sum_{j=1}^{m} c_i q_{ij}(t) < -\alpha(t), t \in R, i = 1, \dots, m,$$

and denote

$$\beta(t) := \min\{\alpha(t), \frac{\ln \gamma}{\tau}\},\ \ \eta_k := 1 + \max_{1\leq i\leq m}\{\sum_{j=1}^{m} c_i u_{ij}(t_k)\}, k \in \mathbb{N}.$$

Then,

i) the zero solution of Eq.(4.1) is uniformly stable if

$$\ln \eta_k \leq \int_{t_{k-1}}^{t_k} \beta(s)ds, k \in \mathbb{N};$$

ii) the zero solution of Eq.(4.1) is globally asymptotically stable if

$$\overline{\lim_{k\to\infty}}\{\ln\eta_k - \int_{t_{k-1}}^{t_k}\beta(s)ds\} < 0,$$
$$\sup_{k\in\mathbb{N}}\{t_k - t_{k-1}\} < \infty;$$

iii) the zero solution of Eq.(4.1) is globally exponentially stable if

$$\sup_{k\in\mathbb{N}}\{\ln\eta_k - \int_{t_{k-1}}^{t_k}\beta(s)ds\} < 0,$$
$$\sup_{k\in\mathbb{N}}\{t_k - t_{k-1}\} < \infty.$$

Proof. For any $\phi \in PC$, let $x(t)$ be a solution of Eq. (4.1) through (t_0, ϕ). No loss of generality, let $t_0 < t_1$. It is easily verified that the following formula for the variation of parameters is valid

$$\begin{aligned}(4.3)\qquad x_i(t) &= C_i(t,t_0)x_i(t_0) + \int_{t_0}^t C_i(t,s)g_i(s,x(s),x(s-\tau_i(s)))ds\\ &\quad + \sum_{t_0<t_k\le t} C_i(t,t_k)I_i(t_k,x(t_k^-)),\ i=1,\dots,m, t\ge t_0.\end{aligned}$$

From (H_1'), (H_2) and (4.3), we have

$$\begin{aligned}\|x_i(t)\| &\le c_ie^{-\int_{t_0}^t p_i(s)ds}\|\phi(0)\| + \int_{t_0}^t c_ie^{-\int_s^t p_i(\xi)d\xi}\|g_i(s,x(s),x(s-\tau_i(s)))\|ds\\ &\quad + \sum_{t_0<t_k\le t} c_ie^{-\int_{t_k}^t p_i(s)ds}\|I_i(t_k,x(t_k^-))\|\\ &\le c_ie^{-\int_{t_0}^t p_i(s)ds}\|\phi(0)\| + \int_{t_0}^t c_ie^{-\int_s^t p_i(\xi)d\xi}\sum_{j=1}^m[l_{ij}(s)\|x_j(s)\|\\ &\quad + q_{ij}(s)\|x_j(s)\|_\tau]ds + \sum_{t_0<t_k\le t}[c_ie^{-\int_{t_k}^t p_i(s)ds}\sum_{j=1}^m u_{ij}(t_k)\|x_j(t_k^-)\|].\end{aligned}$$

Denote

$$\begin{aligned}z_i(t) &= c_ie^{-\int_{t_0}^t p_i(s)ds}\|\phi(0)\| + \int_{t_0}^t c_ie^{-\int_s^t p_i(\xi)d\xi}\sum_{j=1}^m[l_{ij}(s)\|x_j(s)\| + q_{ij}(s)\|x_j(s)\|_\tau]ds\\ &\quad + \sum_{t_0<t_k\le t}[c_ie^{-\int_{t_k}^t p_i(s)ds}\sum_{j=1}^m u_{ij}(t_k)\|x_j(t_k^-)\|],\ t\ge t_0,\\ z_i(t) &= c_i\|\phi(t-t_0)\|,\quad t_0-\tau\le t\le t_0,\ i=1,\dots,m.\end{aligned}$$

Then,

$$(4.4)\qquad \begin{cases} D^+z_i(t) & \le -p_i(t)z_i(t) + \sum_{j=1}^m c_il_{ij}(t)z_j(t) + \sum_{j=1}^m c_iq_{ij}(t)[z_j(t)]_\tau,\quad t\ne t_k,\\ z_i(t_k) & \le z_i(t_k^-) + \sum_{j=1}^m c_iu_{ij}(t_k)z_j(t_k^-),\quad t\ge t_0,\quad k\in\mathbb{N}.\end{cases}$$

Since $\beta(t)=\min\{\alpha(t),\frac{\ln\gamma}{\tau}\}\geq 0$, we have

$$\sup_{\theta\in[-\tau,0]}\{\int_{t+\theta}^{t}\beta(s)ds\}\leq\int_{t-\tau}^{t}\frac{\ln\gamma}{\tau}ds=\ln\gamma.$$

Thus, from (H_4)

$$\begin{aligned}(4.5)\qquad & -p_i(t)+\sum_{j=1}^{m}c_il_{ij}(t)+\sum_{j=1}^{m}c_iq_{ij}(t)e^{\sup_{\theta\in[-\tau,0]}\{\int_{t+\theta}^{t}\beta(s)ds\}}\\ & \leq -p_i(t)+\sum_{j=1}^{m}c_il_{ij}(t)+\gamma\sum_{j=1}^{m}c_iq_{ij}(t)\\ & < -\alpha(t)\leq-\beta(t),\ \ t\geq t_0, i=1,\ldots,m.\end{aligned}$$

Let $\eta_0=1$ and $c=\max_{1\leq i\leq m}\{c_i\}\geq 1$. Next, we shall prove that

$$(4.6)\qquad z_i(t)\leq\eta_0\eta_1\ldots\eta_{k-1}c\|\phi\|_\tau e^{-\int_{t_0}^{t}\beta(s)ds},\ \ t_{k-1}\leq t<t_k, k\in\mathbb{N}.$$

Since $z_i(t)\leq c\|\phi\|_\tau e^{-\int_{t_0}^{t}\beta(s)ds}$ for $t_0-\tau\leq t\leq t_0$, by (4.4), (4.5) and Lemma 2.1, we can get

$$z_i(t)\leq c\|\phi\|_\tau e^{-\int_{t_0}^{t}\beta(s)ds}, t_0\leq t<t_1.$$

Suppose that for $k=1,\ldots,l$

$$z_i(t)\leq\eta_0\eta_1\ldots\eta_{k-1}c\|\phi\|_\tau e^{-\int_{t_0}^{t}\beta(s)ds}, t_{k-1}\leq t<t_k.$$

Then, from (4.4),

$$\begin{aligned}z_i(t_l) &\leq z_i(t_l^-)+\sum_{j=1}^{m}c_iu_{ij}(t_l)z_j(t_l^-)\\ &\leq [1+\sum_{j=1}^{m}c_iu_{ij}(t_l)]\eta_0\ldots\eta_{l-1}c\|\phi\|_\tau e^{-\int_{t_0}^{t_l}\beta(s)ds}\\ &\leq \eta_0\ldots\eta_{l-1}\eta_lc\|\phi\|_\tau e^{-\int_{t_0}^{t_l}\beta(s)ds},\end{aligned}$$

and so

$$z_i(t)\ \ \leq\ \ \eta_0\ldots\eta_{l-1}\eta_lc\|\phi\|_\tau e^{-\int_{t_0}^{t}\beta(s)ds},\ t_l-\tau\leq t\leq t_l.$$

Using Lemma 2.1, we obtain

$$z_i(t)\leq\eta_0\ldots\eta_{l-1}\eta_lc\|\phi\|_\tau e^{-\int_{t_0}^{t}\beta(s)ds},\ t_l\leq t<t_{l+1}.$$

By the induction, the estimate (4.6) holds. Accordingly,

$$(4.7)\qquad \|x_i(t)\|\leq z_i(t)\leq\eta_0\ldots\eta_{k-1}c\|\phi\|_\tau e^{-\int_{t_0}^{t}\beta(s)ds},\ t_{k-1}\leq t<t_k, k\in\mathbb{N}.$$

Noting $\int_{t_{k-1}}^{t_k}(-\beta(s))^+ds=0<\infty$, we can complete the rest of the proof in a similar way as in one of Theorem 3.1 and omit it. □

In the following, we shall discuss the stabilization problem of the large-scale dynamical system with delays

$$(4.8) \qquad \dot{x}_i(t) = A_i x_i(t) + g_i(t, x(t), x(t-\tau_i(t))), i = 1, \dots, m,$$

by introducing the linear impulses

$$(4.9) \qquad \triangle x_i(t_k) = B_{ik} x_i(t_k^-),\ k \in \mathbb{N}.$$

Theorem 4.2. *Assume that (H_1') with $l_{ij}(t) = l_{ij}$ and $q_{ij}(t) = q_{ij}$ holds. Then the zero solution of (4.8), (4.9) is globally exponentially stable provided that*

$$(4.10) \qquad \frac{\ln \delta_i}{\rho} < -\mu_i^+ - \frac{1}{\delta_i}\sum_{j=1}^{m}[l_{ij} + q_{ij}], \quad i = 1, \dots, m,$$

where

$$(4.11) \qquad \delta_i := \sup_{k\in\mathbb{N}}\{\|E + B_{ik}\|\} > 0,\ \rho := \sup_{k\in\mathbb{N}}\{t_k - t_{k-1}\},$$

$$(4.12) \qquad \mu_i := \max_{1\le l\le n_i}\{a_{ll}^{(i)} + \sum_{j=1, j\neq l}^{n_i}|a_{jl}^{(i)}|\}.$$

Proof. From (4.10) and $l_{ij}, q_{ij} \geq 0$, we have

$$\ln \delta_i + \mu_i^+ \rho < 0, i = 1, \dots, m.$$

Denote $\lambda_i := -\ln \delta_i - \mu_i^+\rho > 0$, then

$$\ln\|E + B_{ik}\| + \int_{t_{k-1}}^{t_k} \mu_i ds \quad \leq \quad \ln \delta_i + \mu_i^+ \rho = -\lambda_i < 0,$$

and

$$\varrho_i = \sup_{k\in\mathbb{N}}\{\int_{t_{k-1}}^{t_k} \mu_i^+ ds\} = \mu_i^+\rho < +\infty,\ i = 1, \dots, m.$$

By Theorem 3.3, we obtain Condition (H_3) with

$$c_i = e^{\lambda_i + \varrho_i} = e^{-\ln \delta_i} = \frac{1}{\delta_i}, \quad p_i = \frac{\lambda_i}{\rho} = -\frac{\ln \delta_i}{\rho} - \mu_i^+, \quad i = 1, \dots, m.$$

From (4.10), we can get

$$-p_i + c_i[\sum_{j=1}^{m} l_{ij} + \sum_{j=1}^{m} q_{ij}] < 0, \qquad i = 1, \dots, m.$$

And so, there must be constants $\gamma > 1$ and $\alpha > 0$ such that (H_4) holds, i.e.,

$$-p_i + c_i[\sum_{j=1}^{m} l_{ij} + \gamma\sum_{j=1}^{m} q_{ij}] < -\alpha, \qquad i = 1, \dots, m.$$

In a similar way as the proof of the estimate (4.7) in Theorem 4.1, we obtain

$$\|x_i(t)\| \leq \eta_0 \dots \eta_{k-1} c\|\phi\|_\tau e^{-\int_{t_0}^t \beta ds}, \qquad t_{k-1} \leq t < t_k, k \in \mathbb{N},$$

where

$$\eta_0 = \cdots = \eta_{k-1} = 1,\ c = \max_{1\le i\le n}\{c_i\},\ \beta = \min\{\alpha, \frac{\ln\gamma}{\tau}\} > 0.$$

Hence, the zero solution of system (4.8) is globally exponentially stable. □

Remark 4.1. According to Theorem 4.2, we can present a simple arithmetic to stabilize large-scale delay system (4.8) by utilizing impulses as the following steps.

1) Calculate the parameters μ_i, l_{ij}, q_{ij} in terms of (H_1') and (4.12).

2) Choose matrices B_{ik} to ensure $\delta_i = \sup_{k\in\mathbb{N}}\{\|E + B_{ik}\|\} < 1$.

3) To ensure exponential stability, take

$$\rho = \sup_{k\in\mathbb{N}}\{t_k - t_{k-1}\} < \min_{1\le i\le m}\{\frac{-\delta_i\ln\delta_i}{\mu_i^+\delta_i + \sum_{j=1}^m(l_{ij}+q_{ij})}\}. \tag{4.13}$$

5. EXAMPLES OF IMPULSIVE STABILIZATION

Example 5.1. Consider an uncertain impulsive large-scale system without delay

$$\begin{cases} \dot{x}_i(t) = A_i(t)x_i(t) + f_i(t, x(t)), & t \ne k\pi,\ t \ge 0, \\ \triangle x_i(t) = B_{ik}x_i(t^-) + I_i(t, x(t^-)), & t = k\pi,\ k \in \mathbb{N}, \end{cases} \tag{5.1}$$

where $i = 1, 2, x_i \in \mathbb{R}^2$, $x = (x_1^T, x_2^T)^T$, $B_{ik} \in \mathbb{R}^{2\times2}$, $I_i \in C[\mathbb{R}^+ \times \mathbb{R}^2, \mathbb{R}^2]$, and

$$A_1 = \begin{pmatrix} 0.1 & 0.1 \\ 0 & -0.2 \end{pmatrix},\ A_2 = \begin{pmatrix} 0 & 0.5 \\ 0.1 & -0.5 \end{pmatrix},$$

$$\begin{aligned} \|f_1(t, x(t))\| &\le \frac{|\sin t|}{10}\|x_1\| + \frac{|\cos t|}{10}\|x_2\|, \\ \|f_2(t, x(t))\| &\le \frac{|\cos t|}{10}\|x_1\| + \frac{|\sin t|}{20}\|x_2\|, \\ \|I_i(t, x)\| &\le u_{i1}(t)\|x_1\| + u_{i2}(t)\|x_2\|. \end{aligned}$$

The corresponding continuous system may be unstable (e.g., the one with $f_1 = f_2 \equiv 0$). According to (3.3), (3.4), we have $\mu_1 = 0.1,\ \mu_2 = 0.1, p(t) = 0.1 + 0.1(|\sin t| + |\cos t|)$, and so $\int_{t_{k-1}}^{t_k} p(s)ds = 0.1\pi + 0.4$. Let

$$\eta_k \le e^{-(0.1\pi+0.4)},\ \text{where } \eta_k = \max_{1\le i\le 2}\{\|E + B_{ik}\| + \sum_{j=1}^{2} u_{ij}(t_k)\}. \tag{5.2}$$

It follows from the case i) in Theorem 3.1 that the zero solution of (5.1) is uniformly stable. For example, taking

$$B_{1k} = B_{2k} = \begin{pmatrix} -0.6 & 0 \\ 0 & -0.6 \end{pmatrix}, I_1(t, x) = I_2(t, x) = 0, \tag{5.3}$$

we easily verify that the condition (5.2) holds. In fact, the zero solution of the impulsive system (5.1) with (5.3) is globally exponentially stable from the case iii) in Theorem 3.1. Therefore, we stabilize the uncertain large-scale system.

Example 5.2. Consider a large-scale dynamical system with delays as follows

$$\begin{cases} \dot{x}_1(t) = A_1 x_1(t) + g_1(t, x(t-\tau(t))), \\ \dot{x}_2(t) = A_2 x_2(t) + g_2(t, x(t-\tau(t))), \end{cases} \tag{5.4}$$

where $t \geq 0$, $x_i = (x_{i1}, x_{i2})^T, i = 1, 2$, $x = (x_1^T, x_2^T)^T$, $\tau(t) = |\cos(t)|$ and

$$A_1 = \begin{pmatrix} 0.5 & 0.1 \\ 0 & 0.4 \end{pmatrix},\ A_2 = \begin{pmatrix} 0.2 & 0 \\ 0.1 & 0.3 \end{pmatrix},$$

$$\begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = \begin{pmatrix} 0 & -0.1 & 0.2 & 0.1 \\ 0.1 & 0 & -0.1 & 0.2 \\ 0.1 & -0.1 & 0.1 & 0 \\ 0 & 0.2 & -0.1 & 0.1 \end{pmatrix} \begin{pmatrix} \sin(x_{11}(t-\tau(t))) \\ x_{12}(t-\tau(t)) \\ \arctan(x_{21}(t-\tau(t))) \\ x_{22}(t-\tau(t)) \end{pmatrix}.$$

Figure 1 shows the zero solution of the continuous system (5.4) is unstable.

According to the steps given in Remark 4.1, we easily stabilize the delay system (5.4) by the impulsive effects

$$\begin{cases} \triangle x_1(t_k) = B_{1k} x_1(t_k^-) \\ \triangle x_2(t_k) = B_{2k} x_2(t_k^-) \end{cases},\ k \in \mathbb{N}. \tag{5.5}$$

Firstly, we work out

$$\mu_1 = 0.5, \mu_2 = 0.3, l_{ij} = 0, q_{11} = 0.1, q_{12} = 0.3, q_{21} = 0.3, q_{22} = 0.2.$$

Next, choose

$$B_{1k} = B_{2k} = \begin{pmatrix} -0.5 & 0 \\ 0 & -0.5 \end{pmatrix},\ k \in \mathbb{N}. \tag{5.6}$$

Clearly, $\delta_1 = \delta_2 = \sup_{k \in \mathbb{N}} \{\|E + B_{ik}\|\} = 0.5 < 1$. In terms of (4.13), we take

$$\rho = \sup_{k \in \mathbb{N}} \{t_k - t_{k-1}\} < 0.5331. \tag{5.7}$$

It follows from Theorem 4.2 that the zero solution of the system (5.4) with impulsive effects (5.5), (5.6), (5.7) is globally exponentially stable.

Figure 2 shows the stability when taking $t_k = 0.5k$ and the initial functions: $x_{11}(t) = \cos(t)$, $x_{12}(t) = \sin(t)$, $x_{21}(t) = -\cos(t)$, $x_{22}(t) = -\sin(t)$, $t \in [-1, 0]$.

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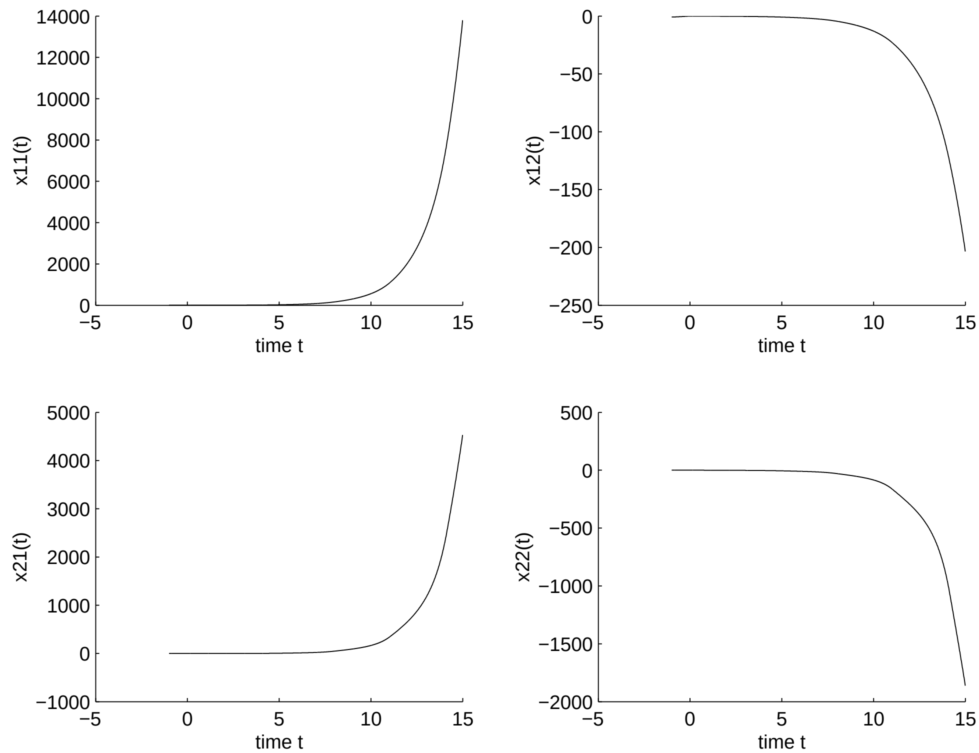

FIGURE 1. Instability of the delay system (5.4) without impulsive effect.

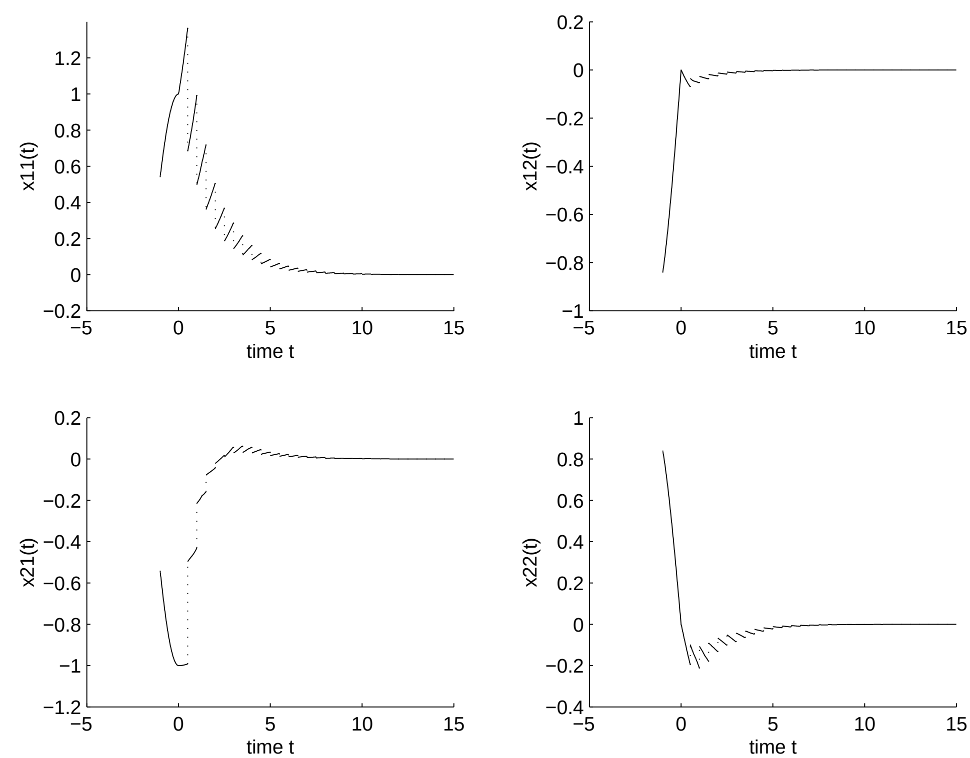

FIGURE 2. Impulsive Stabilization of the delay system (5.4).

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