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ASYMPTOTIC STABILITY OF LINEAR STATE-DELAYED NEUTRAL SYSTEMS WITH POLYTOPE TYPE UNCERTAINTIES

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ABSTRACT. In this paper, a class of linear state-delayed neutral systems with polytope type uncertainties is studied. Using an improved Lyapunov Krasovskii parameter-dependent functional and linear matrix inequality (LMI) technology, new delay-dependent sufficient conditions for the asymptotic stability of the system are first established in terms of Mondie-Kharitonov type's LMI conditions.

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1. INTRODUCTION

Stability analysis of linear neutral systems has received much attention in the past decades, e.g. see [1, 2, 3, 11] and the references therein. Theoretically, the linear neutral system with time delays is much more complicated, especially for the case where the system matrices belong to some convex polytope [9]. By using parameter-dependent Lyapunov functionals, some less conservative results for asymptotic stability of uncertain polytopic delay systems have been proposed in [5, 10] via LMIs. Although these results improve the estimate of asymptotic stability domain, some conservatism still remain since common matrix variable required to satisfy the whole sets of LMIs. An LMI-based method is proposed in [8, 12] for asymptotic stability of neutral systems with time delays, but the polytope uncertainties are not taken into account therein. To the best of our knowledge, so far, no result on the stability for linear neutral state-delayed systems with polytope uncertainties is available in the literature, which is still open and remains unsolved. This motivates our present investigation.

In this paper, we study asymptotic stability of linear neutral time-delay systems with polytope uncertainties. The novel feature of the results obtained in this paper in comparison with [5, 8, 10, 12] is twofold. First, the neutral system considered in this paper is convex polytopic uncertain subjected to state delay. Second, by employing an improved parameter-dependent Lyapunov Krasovskii functional and LMI technology, delay-dependent sufficient conditions for the asymptotic stability of the system are first obtained in terms of Mondie-Kharitonov type's LMIs [7], which can be effectively solved by various convex optimization algorithms and LMI Toolbox of Matlab.

The paper is organized as follows: Section 2 presents notations, definitions and some well-known technical propositions needed for the proof of the main result. Delay-dependent stability conditions of the system is presented in Section 3 with numerical example. The paper ends with conclusions and cited references.

2. PRELIMINARIES

The following notations and definitions will be employed throughout this paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the n-dimensional Euclidean space with the scalar product $\langle \cdot, \cdot \rangle$; $R^{n\times r}$ denotes the space of all matrices of $(n \times r)$-dimensions; A^T denotes the transpose of A; $\lambda(A)$ denotes the set of all eigenvalues of A, $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$, $\lambda_{\min}(A) = \min\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; matrix $Q \geq 0$ ($Q > 0$, resp.) means Q is semi-positive definite matrix i.e. $\langle Qx, x\rangle \geq 0$, $\forall x \in R^n$ (positive definite, resp. i.e. $\langle Qx, x\rangle > 0, \forall x \in R^n, x \neq 0$), $A \geq B$ means $A - B \geq 0$; $C([a, b], R^n)$ denotes the set of all R^n-valued continuous functions on $[a, b]$; the segment of the trajectory $x(t)$ is denoted by $x_t = \{x(t+s) : s \in t \in [-h, 0]\}$ with its norm $\|x_t\| = \sup_{s\in[-h,0]} \|x)(+s)\|$.

Consider the following linear polytopic neutral system with state delay:

$$(2.1)\qquad \begin{cases} \dot{x}(t) - C(\xi)\dot{x}(t-\tau_2) = A_0(\xi)x(t) + A_1(\xi)x(t-\tau_1) \quad t \geq 0 \\ x(\theta) = \varphi(\theta), \quad \forall \theta \in [-h, 0] \end{cases}$$

where $x(t) \in R^n$ is the state, τ_1, τ_2 are time delays, $h = \max\{\tau_1, \tau_2\}$. The state-space data are subject to uncertainties and belong to the polytope Ω given by

$$\Omega = \left\{ [C, A_0, A_1](\xi) := \sum_{i=1}^{p} \xi_i [C_i, A_{0i}, A_{1i}], \quad \sum_{i=1}^{p} \xi_i = 1, \xi_i \geq 0 \right\},$$

where C_i, A_{0i}, A_{1i}, $(i = 1, \ldots, p)$ are constant matrices with appropriate dimensions and ξ_i, $(i = 1, \ldots, p)$ are time-invariant uncertainties.

Let

$$||x_t||_w = \sqrt{||x(t)||^2 + \int_{-h}^{0} ||\dot{x}(t+s)||^2 ds}.$$

Proposition 2.1. [6] *Assume* $||C_i|| < 1, i = 1, 2, \ldots, p$. *If there is a Lyapunov-Krasovskii functional* $V(t, x_t)$ *satisfying*

i) $\exists c_1, c_2 > 0 \quad c_1||x(t)||^2 \leq V(t, x_t) \leq c_2||x_t||_w^2$,

ii) $\exists c_3 > 0 \quad \dot{V}(t, x_t) < -c_3||x(t)||^2$, *for all solution $x(t)$ of the system, then the zero solution of system (2.1) is asymptotically stable.*

Proposition 2.2. [4] *For any symmetric positive definite $M \in R^{n\times n}$, and number $\sigma > 0$, vector function $w : [0,\sigma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, we have*

$$\Big(\int_0^\sigma w(s)ds\Big)^T M\Big(\int_0^\sigma w(s)ds\Big) \le \sigma \int_0^\sigma w^T(s)Mw(s)ds.$$

3. MAIN RESULT

Let $Q_{1i}, Q_{2i}, Q_{3i}, Q_{4i}, Q_{5i}, P_i, T_i, H_i, S_i, M$ be $n\times n$ matrices. We denote

$$\Xi_i(P_j,T_j,H_j,S_j,K_j,\mathcal{Q}_j) =: \begin{bmatrix} \Xi_{11} & A_{0i}^TQ_{2j}^T & \Xi_{13} & -Q_{1j}+A_{0i}^TQ_{4j}^T & \Xi_{15} \\ \star & -S_j & Q_{2j}A_{1i} & -Q_{2j} & Q_{2j}C_i \\ \star & \star & \Xi_{33} & A_{1i}^TQ_{4j}-Q_{3j} & \Xi_{35} \\ \star & \star & \star & \Xi_{44} & -Q_{5j}^T+Q_{4j}C_i \\ \star & \star & \star & \star & \Xi_{55} \end{bmatrix},$$

where

$\Xi_{11} = (P_j+Q_{1j})A_{0i} + A_{0i}^T(P_j+Q_{1j}^T) + S_j - H_j + K_j,$
$\Xi_{13} = H_j + (P_j+Q_{1j})A_{1i} + A_{0i}^TQ_{3j},$
$\Xi_{15} = (P_j+Q_{1j})C_i + A_{0i}^TQ_{5j}^T,$
$\Xi_{33} = -H_j + Q_{3j}A_{1i} + A_{1i}^TQ_{3j}^T - K_j,$
$\Xi_{35} = A_{1i}^TQ_{5j}^T + Q_{3j}C_i,$
$\Xi_{44} = T_j + \tau_1^2H_j - Q_{4j} - Q_{4j}^T,$
$\Xi_{55} = -T_j + Q_{5j}C_i + C_i^TQ_{5j}^T,$

$$\mathbb{M} = \begin{pmatrix} M & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$\lambda_1 = \min_{i=1,2,\dots,p}\lambda_{\min}(P_i), \quad \alpha_P = \max_{i=1,2,\dots,p}\lambda_{\max}(P_i), \quad \alpha_T = \max_i\lambda_{\max}(T_i),$
$\alpha_S = \max_i\lambda_{\max}(S_i), \quad \alpha_K = \max_i\lambda_{\max}(K_i), \quad \alpha_H = \max_i\lambda_{\max}(H_i),$
$\lambda_2 = \alpha_P + \alpha_T + 2h(1+h)\alpha_S + 2h(1+h)\alpha_K + h^2\alpha_H.$

Theorem 3.1. *Assume that $||C_i|| < 1, (i=1,\dots,p)$. The zero solution of system (2.1) is asymptotically stable if there exist matrices $Q_{1i}, Q_{2i}, Q_{3i}, Q_{4i}, Q_{5i} (i=1,\dots,p)$, a symmetric semi-positive definite matrix M and symmetric positive definite matrices P_i, T_i, H_i, S_i, K_i $(i=1,\dots,p)$, such that the following linear matrix inequalities hold:*

$$\Xi_i(P_i,T_i,H_i,S_i,K_i,\mathcal{Q}_i) < -\mathbb{M}, \quad i=1,\dots,p \tag{3.1}$$

$$\Xi_i(P_j,T_j,H_j,S_j,K_j,\mathcal{Q}_j)+\Xi_j(P_i,T_i,H_i,S_i,K_i,\mathcal{Q}_i)<\frac{2}{p-1}\mathbb{M}, \tag{3.2}$$

$i=1,\dots,p-1, j=i+1,\dots,p.$

Proof. We denote $[P,T,S,M,H,K,Q_k](\xi):=\sum_{i=1}^{p}\xi_i[P_i,T_i,S_i,M_i,H_i,K_i,Q_{ki}]$, and consider the following Lyapunov functional:

$$V(t,x_t)=V_1(\cdot)+V_2(\cdot)+V_3(\cdot)+V_4(\cdot)+V_5(\cdot),$$

where
$V_1(\cdot)=x^T(t)P(\xi)x(t)$,
$V_2(\cdot)=\int_{t-\tau_2}^{t}\dot{x}^T(s)T(\xi)\dot{x}(s)ds$,
$V_3(\cdot)=\int_{t-\tau_2}^{t}x^T(s)S(\xi)x(s)ds$,
$V_4(\cdot)=\int_{t-\tau_1}^{t}x^T(s)K(\xi)x(s)ds$,
$V_5(\cdot)=\tau_1\int_{t-\tau_1}^{t}(s-(t-\tau_1))\dot{x}^T(s)H(\xi)\dot{x}(s)ds$. It is easy to verify that

$$\lambda_1||x(t)||^2\le V(t,x_t)\le\lambda_2||x_t||_w^2. \tag{3.3}$$

The derivatives of $V_i(.), i=1,2,\dots,5$ along the trajectory of system (2.1) are given by

$$\begin{aligned}\dot{V}_1(t)&=2x(t)^TP(\xi)\dot{x}(t)\\&=2x(t)^TP(\xi)[A_0(\xi)x(t)+A_1(\xi)x(t-\tau_1)+C(\xi)\dot{x}(t-\tau_2)]\\&=\sum_{i=1}^{p}\xi_i\Big[\sum_{j=1}^{p}\xi_j\Big(x^T(t)(P_iA_{0j}+A_{0j}^TP_i)x(t)+2x^T(t)P_iA_{1j}x(t-\tau_1)\\&\quad+2x^T(t)P_iC_jx(t-\tau_2)\Big)\Big],\end{aligned} \tag{3.4}$$

$$\begin{aligned}\dot{V}_2(t)&=\dot{x}^T(t)T(\xi)\dot{x}(t)-\dot{x}^T(t-\tau_2)T(\xi)\dot{x}(t-\tau_2)\\&=\sum_{i=1}^{p}\xi_i\Big[\sum_{j=1}^{p}\xi_j\Big(\dot{x}^T(t)T_i\dot{x}(t)-\dot{x}^T(t-\tau_2)T_i\dot{x}(t-\tau_2)\Big)\Big],\end{aligned} \tag{3.5}$$

$$\begin{aligned}\dot{V}_3(t)&=x^T(t)S(\xi)x(t)-x^T(t-\tau_2)S(\xi)x(t-\tau_2)\\&=\sum_{i=1}^{p}\xi_i\Big[\sum_{j=1}^{p}\xi_j\Big(x^T(t)S_ix(t)-x^T(t-\tau_2)S_ix(t-\tau_2)\Big)\Big],\end{aligned} \tag{3.6}$$

$$\begin{aligned}\dot{V}_4(t)&=x^T(t)K(\xi)x(t)-x^T(t-\tau_1)K(\xi)x(t-\tau_1)\\&=\sum_{i=1}^{p}\xi_i\Big[\sum_{j=1}^{p}\xi_j\Big(x^T(t)K_ix(t)-x^T(t-\tau_1)K_ix(t-\tau_1)\Big)\Big].\end{aligned} \tag{3.7}$$

Using Proposition 2.2, we have

$$\dot{V}_5(t)={\tau_1}^2\dot{x}^T(t)H(\xi)\dot{x}(t)-\tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)H(\xi)\dot{x}(s)ds$$

$$
\begin{aligned}
&\leq {\tau_1}^2\dot{x}^T(t)H(\xi)\dot{x}(t) - \Big(\int_{t-\tau_1}^{t}\dot{x}^T(s)ds\Big)^T H(\xi)\Big(\int_{t-\tau_1}^{t}\dot{x}^T(s)ds\Big)\\
&= {\tau_1}^2\dot{x}^T(t)H(\xi)\dot{x}(t) - x^T(t)H(\xi)x(t) + 2x^T(t)H(\xi)x(t-\tau_1)\\
&\qquad - x^T(t-\tau_1)H(\xi)x(t-\tau_1)\\
&= \sum_{i=1}^{p}\xi_i\Big[\sum_{j=1}^{p}\xi_j\Big({\tau_1}^2\dot{x}^T(t)H_i\dot{x}(t) - x^T(t)H(\xi)x(t) + 2x^T(t)H_ix(t-\tau_1)\\
&\qquad - x^T(t-\tau_1)H_ix(t-\tau_1)\Big)\Big]. \qquad (3.8)
\end{aligned}
$$

Since $-\dot{x}(t) + C(\xi)\dot{x}(t-\tau_2) + A_0(\xi)x(t) + A_1(\xi)x(t-\tau_1) = 0$, we have

$$
\begin{aligned}
&2[x^T(t)Q_1(\xi) + x^T(t-\tau_2)Q_2(\xi) + x^T(t-\tau_1)Q_3(\xi) + \dot{x}^T(t)Q_4(\xi) + \dot{x}^T(t-\tau_2)Q_5(\xi)]\\
&\qquad \times [-\dot{x}(t) + C(\xi)\dot{x}(t-\tau_2) + A_0(\xi)x(t) + A_1(\xi)x(t-\tau_1)] = 0.
\end{aligned}
$$

From (3.4)–(3.8) it follows that

$$
\begin{aligned}
\dot{V}(t,x_t) \leqslant &\sum_{i=1}^{p}\xi_i\Big[\sum_{j=1}^{p}\xi_j\Big(x^T(t)[P_iA_{0j} + A_{0j}^TP_i + S_i - H_i + K_i + Q_{1i}A_{0j} + A_{0j}^TQ_{1i}^T]x(t)\\
&- 2x^T(t)A_{0j}^TQ_{2i}^Tx(t-\tau_2) + 2x^T(t)[P_iA_{1j} + Q_{1i}A_{1j} + A_{0j}^TQ_{3i}^T]x(t-\tau_1)\\
&+ 2x^T(t)[-Q_{1i} + A_{0j}^TQ_{4i}^T]\dot{x}(t) + 2x^T(t)[P_iC_j + Q_{1i}C_j + A_{0j}^TQ_{5i}^T]\dot{x}(t-\tau_2)\\
&- x^T(t-\tau_2)S_ix(t-\tau_2) + 2x^T(t-\tau_2)Q_{2i}A_{1j}(t-\tau_1) - 2x^T(t-\tau_2)Q_{2i}\dot{x}(t)\\
&+ x^T(t-\tau_2)Q_{2i}\dot{x}(t-\tau_2) + x^T(t-\tau_1)[-H_i - K_i + Q_{3i}A_{1j} + A_{1j}^TQ_{3i}^T]x(t-\tau_1)\\
&+ 2x^T(t-\tau_1)(A_{1j}^TQ_{4i}^T - Q_{3i})\dot{x}(t) + 2x^T(t-\tau_1)(A_{1j}^TQ_{5i}^T + Q_{3i}C_j)\dot{x}(t-\tau_2)\\
&+ \dot{x}^T(t)(T_i + \tau_1^2H_i - Q_{4i} - Q_{4i}^T)\dot{x}(t) + 2\dot{x}^T(t)(-Q_{5i}^T + Q_{4i}C_j)\dot{x}(t-\tau_2)\\
&+ \dot{x}^T(t-\tau_2)(-T_i + Q_{5i}C_j + C_j^TQ_{5i}^T)\dot{x}(t-\tau_2)\Big)\Big]\\
= &\sum_{i=1}^{p}\xi_i^2\Big[x^T(t)[P_iA_{0i} + A_{0i}^TP_i + S_i - H_i + K_i + Q_{1i}A_{0i} + A_i^TQ_{1i}^T]x(t)\\
&- 2x^T(t)A_{0i}^TQ_{2i}^Tx(t-\tau_2) + 2x^T(t)[P_iA_{1i} + Q_{1i}A_{1i} + A_{0i}^TQ_{3i}^T]x(t-\tau_1)\\
&+ 2x^T(t)[-Q_{1i} + A_{0i}^TQ_{4i}^T]\dot{x}(t) + 2x^T(t)[P_iC_i + Q_{1i}C_i + A_{0i}^TQ_{5i}^T]\dot{x}(t-\tau_2)\\
&- x^T(t-\tau_2)S_ix(t-\tau_2) + 2x^T(t-\tau_2)Q_{2i}A_{1i}(t-\tau_1) - 2x^T(t-\tau_2)Q_{2i}\dot{x}(t)\\
&+ x^T(t-\tau_2)Q_{2i}\dot{x}(t-\tau_2) + x^T(t-\tau_1)[-H_i - K_i + Q_{3i}A_{1i} + A_{1i}^TQ_{3i}^T]x(t-\tau_1)\\
&+ 2x^T(t-\tau_1))(A_{1i}^TQ_{4i}^T - Q_{3i})\dot{x}(t) + 2x^T(t-\tau_1)(A_{1i}^TQ_{5i}^T + Q_{3i}C_i)\dot{x}(t-\tau_2)\\
&+ \dot{x}^T(t)(T_i + \tau_1^2H_i - Q_{4i} - Q_{4i}^T)\dot{x}(t) + 2\dot{x}^T(t)(-Q_{5i}^T + Q_{4i}C_i)\dot{x}(t-\tau_2)\\
&+ \dot{x}^T(t-\tau_2)(-T_i + Q_{5i}C_i + C_i^TQ_{5i}^T)\dot{x}(t-\tau_2)\Big]\\
&+ \sum_{i=1}^{p}\sum_{j=i+1}^{p-1}\xi_i\xi_j\Big[\Big(x^T(t)[P_iA_{0j} + A_{0j}^TP_i + S_i - H_i + K_i + Q_{1i}A_{0j} + A_j^TQ_{1i}^T]x(t)
\end{aligned}
$$

$$
\begin{aligned}
&- 2x^T(t)A_j^TQ_{2i}^Tx(t-\tau_2) + 2x^T(t)[P_iA_{1j} + Q_{1i}A_{1j} + A_{0j}^TQ_{3i}^T]x(t-\tau_1)\\
&+ 2x^T(t)[-Q_{1i} + A_{0j}^TQ_{4i}^T]\dot{x}(t) + 2x^T(t)[P_iC_j + Q_{1i}C_j + A_j^TQ_{5i}^T]\dot{x}(t-\tau_2)\\
&- x^T(t-\tau_2)S_ix(t-\tau_2) + 2x^T(t-\tau_2)Q_{2i}A_{1j}(t-\tau_1) - 2x^T(t-\tau_2)Q_{2i}\dot{x}(t)\\
&+ x^T(t-\tau_2)Q_{2i}\dot{x}(t-\tau_2) + x^T(t-\tau_1)[-H_i - K_i + Q_{3i}A_{1j} + A_{1j}^TQ_{3i}^T]x(t-\tau_1)\\
&+ 2x^T(t-\tau_1)(B_j^TQ_{4i}^T - Q_{3i})\dot{x}(t) + 2x^T(t-\tau_1)(B_j^TQ_{5i}^T + Q_{3i}C_j)\dot{x}(t-\tau_2)\\
&+ \dot{x}^T(t)(T_i + \tau_1^2H_i - Q_{4i} - Q_{4i}^T)\dot{x}(t) + 2\dot{x}^T(t)(-Q_{5i}^T + Q_{4i}C_j)\dot{x}(t-\tau_2)\\
&+ \dot{x}^T(t-\tau_2)(-T_i + Q_{5i}C_j + C_j^TQ_{5i}^T)\dot{x}(t-\tau_2)\Big)\\
&+ \Big(x^T(t)[P_jA_{0i} + A_{0i}^TP_j + S_j - H_j + K_j + Q_{1j}A_{0i} + A_{0i}^TQ_{1j}^T]x(t)\\
&- 2x^T(t)A_{0i}^TQ_{2j}^Tx(t-\tau_2) + 2x^T(t)[P_jA_{1i} + Q_{1j}A_{1i} + A_{0i}^TQ_{3j}^T]x(t-\tau_1)\\
&+ 2x^T(t)[-Q_{1j} + A_{0i}^TQ_{4j}^T]\dot{x}(t) + 2x^T(t)[P_iC_i + Q_{1j}C_i + A_{0i}^TQ_{5j}^T]\dot{x}(t-\tau_2)\\
&- x^T(t-\tau_2)S_jx(t-\tau_2) + 2x^T(t-\tau_2)Q_{2j}A_{1i}(t-\tau_1(t)) - 2x^T(t-\tau_2)Q_{2j}\dot{x}(t)\\
&+ x^T(t-\tau_2)Q_{2j}\dot{x}(t-\tau_2) + x^T(t-\tau_1)[-H_j - K_j + Q_{3j}A_{1i} + A_{1i}^TQ_{3j}^T]x(t-\tau_1)\\
&+ 2x^T(t-\tau_1))(A_{1i}^TQ_{4j}^T - Q_{3j})\dot{x}(t) + 2x^T(t-\tau_1)(A_{1i}^TQ_{5j}^T + Q_{3j}C_i)\dot{x}(t-\tau_2)\\
&+ \dot{x}^T(t)(T_j + \tau_1^2H_j - Q_{4j} - Q_{4j}^T)\dot{x}(t) + 2\dot{x}^T(t)(-Q_{5j}^T + Q_{4j}C_i)\dot{x}(t-\tau_2)\\
&+ \dot{x}^T(t-\tau_2)(-T_j + Q_{5j}C_i + C_i^TQ_{5j}^T)\dot{x}(t-\tau_2)\Big)\Big].
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\dot{V}(t,x_t) \leq& \eta^T(t)\Big[\sum_{i=1}^{p}\xi_i^2\Xi_i(P_i,T_i,H_i,S_i,K_i,\mathcal{Q}_i)\\
&+\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}\xi_i\xi_j\Big(\Xi_i(P_j,T_j,H_j,S_j,K_j,\mathcal{Q}_j)+\Xi_j(P_i,T_i,H_i,S_i,K_i,\mathcal{Q}_i)\Big)\Big]\eta(t),
\end{aligned}
$$

where $\eta^T(t) = [\ x^T(t)\ \ x^T(t-\tau_2)\ \ x^T(t-\tau_1)\ \ \dot{x}^T(t)\ \ \dot{x}^T(t-\tau_2)]$.

From the conditions (3.1), we can choose a small positive number δ such that

$$
\Xi_i(P_i,T_i,H_i,S_i,K_i,\mathcal{Q}_i) < -\mathbb{M} - \delta I, \quad i = 1,\ldots,p,
$$

and hence

$$
\text{(3.9)}\quad \dot{V}(t,x_t) \leqslant \eta^T(t)\Big(-\sum_{i=1}^{p}\xi_i^2 + \frac{2}{p-1}\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}\xi_i\xi_j\Big)\mathbb{M}\eta(t) - \delta\left(\sum_{i=1}^{p}\xi_i^2\right)||\eta(t)||^2
$$

Observe that

$$
(p-1)\sum_{i=1}^{p}\xi_i^2 - 2\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}\xi_i\xi_j = \sum_{i=1}^{p-1}\sum_{j=i+1}^{p}(\xi_i-\xi_j)^2 \geqslant 0,
$$

and

$$
1 = \left(\sum_{i=1}^{p}\xi_i\right)^2 \leqslant p\left(\sum_{i=1}^{p}\xi_i^2\right),
$$

from (3.9) we obtain

$$\dot{V}(t,x_t) < -\frac{\delta}{p}||\eta(t)||^2 \leq -\frac{\delta}{p}||x(t)||^2.$$

Since $||C_i|| < 1,\ i = 1,2,\dots,p$, by Proposition 2.1, the zero solution of system (2.1) is asymptotically stable. This completes the proof of the theorem. □

Remark 3.1. It is worth noting that the condition (3.1) means the asymptotic stability of each $i^{th}-$subsystem, while the condition (3.2) implies the asymptotic stability of the $ij^{th}-$subsystems and if $p = 1$ this condition is automatically removed. Thus, Theorem 3.1 includes the result of [3, 8, 12] for the neutral systems without polytope type uncertainties ($p = 1$) and of [5, 10] for polytopic time-delay nominal systems ($C = 0$).

An example. Consider system (2.1), where $p = 3, \tau_1 = 3.294, \tau_2 = 2.325$ and

$$A_{01} = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.09 \end{bmatrix},\ A_{02} = \begin{bmatrix} -2 & -1 \\ 0 & -1 \end{bmatrix},\ A_{03} = \begin{bmatrix} -1.9 & 0 \\ 0 & -1 \end{bmatrix},$$

$$A_{11} = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix},\ A_{12} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$A_{13} = \begin{bmatrix} -0.9 & 0 \\ -1 & -1.1 \end{bmatrix}, C_1 = C_2 = C_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

Using Matlab's LMI Toolbox, the LMIs (3.1), (3.2) are feasible with

$$P_1 = 10^3 \times \begin{pmatrix} 1.7286 & 0.0713 \\ 0.0713 & 0.1044 \end{pmatrix}, \quad P_2 = 10^3 \times \begin{pmatrix} 1.7286 & 0.0713 \\ 0.0713 & 0.1044 \end{pmatrix},$$

$$P_3 = 10^3 \times \begin{pmatrix} 1.7286 & 0.0713 \\ 0.0713 & 0.1044 \end{pmatrix}, \quad T_1 = \begin{pmatrix} 340.9984 & -4.0051 \\ -4.0051 & 32.0171 \end{pmatrix},$$

$$T_2 = \begin{pmatrix} 131.1557 & 93.4328 \\ 93.4328 & 267.9493 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 565.8401 & 14.7002 \\ 14.7002 & 14.2196 \end{pmatrix},$$

$$S_1 = \begin{pmatrix} 10.9018 & 0.5682 \\ 0.5682 & 0.3740 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 102.6679 & 27.7092 \\ 27.7092 & 12.5271 \end{pmatrix},$$

$$S_3 = \begin{pmatrix} 22.0684 & -0.0155 \\ -0.0155 & 0.0003 \end{pmatrix}, \quad K_1 = \begin{pmatrix} 126.6476 & 34.8895 \\ 34.8895 & 11.6732 \end{pmatrix},$$

$$K_2 = 10^3 \times \begin{pmatrix} 1.9646 & 0.8557 \\ 0.8557 & 1.0108 \end{pmatrix}, \quad K_3 = 10^3 \times \begin{pmatrix} 5.5205 & 0.1423 \\ 0.1423 & 0.0823 \end{pmatrix},$$

$$H_1 = \begin{pmatrix} 62.9216 & -8.3560 \\ -8.3560 & 6.5904 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 2.5767 & -2.1348 \\ -2.1348 & 61.3053 \end{pmatrix},$$

$$H_3 = \begin{pmatrix} 75.1692 & 2.4741 \\ 2.4741 & 5.0096 \end{pmatrix}, \quad Q_{11} = \begin{pmatrix} -229.3060 & -251.1666 \\ 122.7765 & 43.9443 \end{pmatrix},$$

$$Q_{12} = \begin{pmatrix} -77.2918 & 484.2981 \\ -944.5658 & -629.9611 \end{pmatrix}, \quad Q_{13} = 10^3 \times \begin{pmatrix} -1.2092 & -0.3569 \\ 0.1922 & -0.0225 \end{pmatrix},$$

$$Q_{21} = Q_{22} = Q_{23} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$Q_{31} = \begin{pmatrix} 316.9512 & 58.0430 \\ 95.2396 & 43.3029 \end{pmatrix}, \quad Q_{32} = \begin{pmatrix} 581.9968 & 407.6488 \\ 327.0204 & -158.0781 \end{pmatrix},$$

$$Q_{33} = \begin{pmatrix} 793.9240 & -145.8548 \\ 208.6832 & -32.9185 \end{pmatrix}, \quad Q_{41} = \begin{pmatrix} 967.8075 & 30.9110 \\ 250.1379 & 191.3986 \end{pmatrix},$$

$$Q_{42} = \begin{pmatrix} 388.0444 & 454.6886 \\ -248.6346 & 604.5765 \end{pmatrix}, \quad Q_{43} = \begin{pmatrix} 938.8036 & -177.1645 \\ 224.9416 & 45.2952 \end{pmatrix},$$

$$Q_{51} = \begin{pmatrix} -17.6502 & -52.4232 \\ 2.8598 & -21.0811 \end{pmatrix}, \quad Q_{52} = \begin{pmatrix} -54.0249 & -36.5439 \\ -24.1424 & 14.0410 \end{pmatrix},$$

$$Q_{53} = \begin{pmatrix} -76.2772 & -18.1835 \\ -23.1784 & -11.2059 \end{pmatrix}, \quad M = \begin{pmatrix} 19.7472 & -0.0147 \\ -0.0147 & 0.0002 \end{pmatrix}.$$

By Theorem 3.1 the zero solution of the system is asymptotically stable.

4. CONCLUSIONS

In this paper we have presented delay-dependent sufficient conditions for the asymptotic stability of linear state-delayed neutral systems with polytope type uncertainties. The conditions are established in terms of Mondie-Kharitonov type's LMI conditions.

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